

IMPROVING HEAT TRANSFER IN STEAM-HEATED FAST ROTATING PAPER DRYING DRUMS

WILFRIED ROETZEL*

Chemical Engineering Research Group, Council for Scientific and Industrial Research,
Pretoria, South Africa

(Received 5 March 1974)

Abstract—Condensate film thickness and, consequently, heat transfer depend on the method of condensate removal. Replacing the conventional rotating condensate syphon by a stationary one with the tip running in a circumferential groove the high pressure drop in the rotating syphon is avoided while film thickness is reduced somewhat.

A more significant increase in heat transfer may be obtained by giving the inside surface in the drum a slight slope. For the case that this conical surface is circularly curved in the axial direction equations are presented for the calculation of local and mean film thicknesses. These show that for the desirable virtually uniform heat flux even with very slight curvatures steam side heat-transfer coefficients can be expected to be several times better than on a cylindrical surface.

The developed equations are useful also for other systems of condensation where the acceleration along the condensate flow path is proportional to the flow length. The equations can also be applied for corresponding cases of free convection and film evaporation.

NOMENCLATURE

A, dimensionless group defined by (5) or (40), respectively;
a, exponent;
B, breadth of flow path;
b, acceleration;
C, D, integration constants;
E, dimensionless group defined by (20) or (41), respectively;
F, dimensionless group defined by (27);
Gr, general Grashof number defined by (12) of [8];
g, gravitational acceleration;
h, latent heat of condensation;
I, numerical value representing a definite integral;
L, total flow length of condensate;
m, local condensate flow rate;
Nu, mean Nusselt number for constant heat flux;
Pr, general Prandtl number defined by (9) of [8];
p, pressure;
q, constant heat flux per unit time and area;
R, radius of curvature in condensate flow direction;
r, mean inside radius of drum or minimal distance of rotating wall from axis of rotation;
T, temperature;

u, dummy variable defined by (32) or (36);
v, dummy variable defined by (33) or (37);
x, variable flow length of condensate film;
y, local thickness of condensate film;
z, substitution variable defined by (15).

Greek symbols

α , mean heat-transfer coefficient for constant heat flux and linear temperature profile in the film;
 Δ , finite difference;
 λ , thermal conductivity of condensate;
 ν , mean kinematic viscosity of condensate;
 ξ , dimensionless flow length of condensate defined by (4);
 ρ , density of condensate;
 ϕ , dimensionless thickness of the condensate film defined by (3);
 ψ , relative dimensionless film thickness defined by (23);
 ω , angular velocity of rotating system.

Subscripts

0, at the point where $\xi = 0$;
 1, at the point where $\xi = 1$;
 ∞ , for $R = \infty$;
m, mean value;
 min, minimal;
n, normal to the wall;
s, at saturation conditions;
u, usual.

*Present address: Bayer AG, Verfahrenstechnik R150, 415 Krefeld 11, F.R. Germany.

1. INTRODUCTION

IN PAPER production drying is economically an important step and in the past significant research and development have been devoted to this operation. Han [1] has presented a detailed review on the present state of the art with reference to all important prior publications.

Considerable effort has been made to improve the heat transfer in fast rotating steam dryers because of its direct effect on the production rate. However, the means are somewhat restricted because a uniform heat flux is desirable and, of course, the manufacturing costs of the drying drums must be kept within reason. Under rimming conditions which are considered in this paper the heat transfer is determined mainly by the method of condensate removal. With a stationary syphon the heat transfer is relatively poor because the gap between the syphon's tip and the moving inside surface cannot be made very small (Fig. 1). A considerable decrease of the gap (however, not to zero)

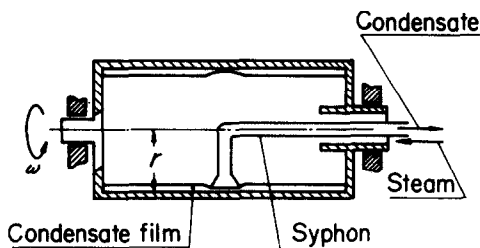


FIG. 1. Conventional steam-heated fast rotating drum.

causing an improvement in heat transfer has been obtained by a rotating syphon with an inlet shoe being held rigidly against the inside dryer surface. The disadvantage of this construction is a high pressure drop caused by the centrifugal forces in the rotating syphon.

In this paper another construction is proposed. The condensate is collected in a circumferential groove in the inside surface and from there it is removed by means of a stationary syphon with its tip in the groove. The high pressure drop of the rotating syphon is now avoided. The mean film thickness is slightly smaller than with a rotating syphon because the condensate is sucked off from a lower level. A further more effective improvement of the heat-transfer coefficient can be obtained by using a conical shape of the inside surface so that the condensate can flow better "down" to the collecting groove. The slope of the cone can be constant or can change according to any desirable function of the flow length, e.g. it can increase proportional to the flow path, which case is being investigated theoretically in this paper (Fig. 2).

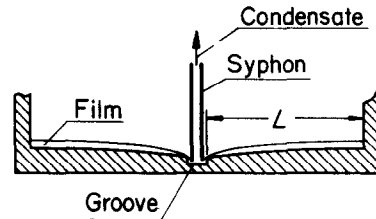


FIG. 2. Drum with two curved conical sections and one sampling groove.

2. DERIVATION OF THE DIFFERENTIAL EQUATION

Figure 3 shows the co-ordinate system used in the subsequent analysis. The co-ordinate x is the variable flow path along the surface. Because of the slope of the surface with respect to the axis of rotation the centrifugal acceleration b forms an angle with the x -axis. Neglecting the gravity forces Fig. 3 is representative of the entire circumference. Assuming a

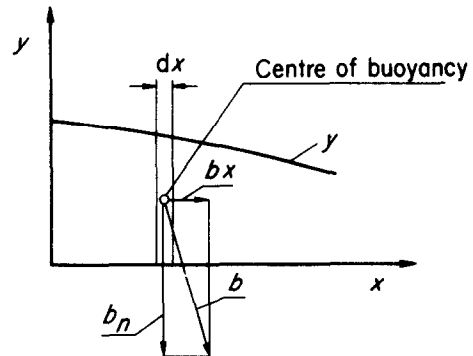


FIG. 3. The condensate film in the acceleration field.

parabolic Nusselt velocity profile in the film, [2] and [3], yields the energy balance

$$b_x \cdot \rho \cdot dx - b_n \cdot \rho \cdot \frac{1}{2} \cdot dy = \frac{3 \cdot \dot{m} \cdot v}{B \cdot y^3} \cdot dx. \quad (1)$$

The mechanical energy per volume produced by moving the centre of buoyancy is transferred to frictional heat (not kinetic energy). The factor $\frac{1}{2}$ in the middle specific energy term takes into account that the centre of buoyancy is situated at one half of the height (y). For $b_n = 0$ (1) is in accordance with equation (12) of [3] where no gravity forces normal to the wall do occur.

The variation of viscosity with temperature can be taken into account by using the reference temperature of Drew and Gregorig (three-quarters of the wall temperature plus one-quarter of the film surface temperature) as shown in [3].

For paper drying a uniform heat flux is desirable

which can be expressed by the local condensate flow rate and the variable flow length

$$\dot{q} = \frac{\dot{m} \cdot h}{B \cdot x} \tag{2}$$

Introducing into (1) the dimensionless film thickness and the dimensionless flow path

$$\phi = \frac{y}{L} \tag{3}$$

$$\xi = \frac{x}{L} \tag{4}$$

as well as the dimensionless group

$$A = \frac{b_n \cdot \rho \cdot h \cdot L^2}{6 \cdot v \cdot \dot{q}} \tag{5}$$

where

$$b_n = \omega^2 \cdot r \tag{6}$$

yields the dimensionless differential equation

$$\frac{d\phi}{d\xi} = 2 \cdot \frac{b_x}{b_n} - \frac{1}{A} \cdot \frac{\xi}{\phi^3} \tag{7}$$

b_x can be any function of the flow path. For our case of constant curvature and small ratios of L/R

$$\frac{b_x}{b_n} = \frac{L}{R} \cdot \xi \tag{8}$$

and b_n can be regarded as constant.

Introducing (8) into (7) yields

$$\frac{d\phi}{d\xi} = \xi \cdot \left(\frac{2 \cdot L}{R} - \frac{1}{A \cdot \phi^3} \right) \tag{9}$$

Separating the variables and integrating gives:

$$\int \frac{d\phi}{\frac{2 \cdot L}{R} - \frac{1}{A \cdot \phi^3}} = \frac{1}{2} \cdot \xi^2 + C. \tag{10}$$

3. INFINITE RADIUS OF CURVATURE

To begin with the limiting case

$$\frac{L}{R} = 0 \tag{11}$$

is considered. The inside surface is exactly cylindrical as in the conventional drying drums. Our case differs from that of a conventional stationary syphon only by the boundary condition of the point $x = L$. Immediately following the edge of the groove the film thickness y is zero and for the calculation it may be assumed $y = 0$ for $x = L$ (especially if the edge if rounded). In the conventional case the film thickness is there equal to the distance of the syphon from the wall.

When a rotating syphon is applied condensate is removed only in one region of the cylinder requiring a multidirectional flow pattern for the condensate. With a stationary syphon in a groove condensate is removed from the whole circumference of the groove. In our case the boundary condition is

$$\xi = 1 \rightarrow \phi = 0. \tag{12}$$

Integrating (10) taking into account (11) and (12) gives

$$\phi = \left(\frac{2}{A} \right)^{1/4} \cdot (1 - \xi^2)^{1/4} \tag{13}$$

which is plotted in Fig. 4.

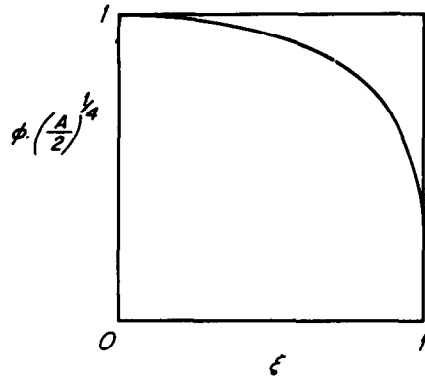


FIG. 4. Local relative film thickness for $L/R = 0$ according to (13).

In the case of non-uniform heat flux but constant temperature difference a mean heat-transfer coefficient, arrived at by an integration of the local coefficient over the area, is usually applied.

In our case of constant heat flux with changing heat-transfer resistance and temperature difference, however, it is more consistent and reasonable to introduce a mean resistance together with a mean temperature difference by integrating both over the area. For this mean resistance the mean film thickness is needed:

$$\phi_m = \int_{\xi=0}^{\xi=1} \phi \cdot d\xi = \left(\frac{2}{A} \right)^{1/4} \cdot I. \tag{14}$$

The definite integral I [defined by (13) and (14)] can be solved by the substitution

$$\xi = \cos z \tag{15}$$

which leads to the following known {see [4], p. 100, equation (43)} integral:

$$I = \int_{z=0}^{z=\pi/2} (\sin z)^{3/2} \cdot dz = \frac{2}{3} \cdot \sqrt{\pi} \cdot \frac{\Gamma^{5/4}}{\Gamma^{3/4}} = 0.87402. \tag{16}$$

Substituting I in (14) according to (16) and introducing the mean Nu number, which is reasonable because a linear temperature profile can be assumed in the film,

$$Nu = \frac{1}{\phi_m} \quad (17)$$

gives for our case of infinite radius of curvature

$$Nu_\infty = (0.86 \cdot A)^{1/4}. \quad (18)$$

This equation is applicable when the heat flux \dot{q} is given. However, frequently the mean temperature difference ΔT_m is given and then the following approach is more convenient. Expressing the heat flux as follows

$$\dot{q} = Nu \cdot \frac{\lambda}{L} \cdot \Delta T_m \quad (19)$$

and defining the dimensionless group

$$E = \frac{b_n \cdot \rho \cdot h \cdot L^3}{6 \cdot v \cdot \lambda \cdot \Delta T_m} \quad (20)$$

gives

$$A = \frac{E}{Nu}. \quad (21)$$

Equations (19)–(21) are valid for any value of R . Replacing A in (18) according to (21) with $Nu = Nu_\infty$ and solving for Nu_∞ yields

$$Nu_\infty = (0.86 \cdot E)^{1/5}. \quad (22)$$

This equation should be used instead of (18), when the mean temperature is given.

4. FINITE RADIUS OF CURVATURE

For simplification of the integration in (10) the relative dimensionless film thickness is introduced

$$\psi = \phi \cdot \left(\frac{2 \cdot L \cdot A}{R} \right)^{1/3} \quad (23)$$

with which (10) turns to

$$\xi^2 + 2 \cdot C = -\frac{R}{L} \cdot \left(\frac{R}{2 \cdot L \cdot A} \right)^{1/3} \cdot \int \frac{\psi^3 \cdot d\psi}{1 - \psi^3}. \quad (24)$$

According to [5] p. 32, Section 2.1.3.1.2, and p. 31, Section 1.2.1, the integration yields

$$\int \frac{\psi^3 \cdot d\psi}{1 - \psi^3} = -\psi + \frac{1}{6} \ln \frac{\psi^2 + \psi + 1}{(\psi - 1)^2} + \frac{1}{\sqrt{3}} \cdot \arctan \frac{2 \cdot \psi + 1}{\sqrt{3}} + D. \quad (25)$$

The integration constants D in (25) and C in (24) must satisfy the boundary condition according to (12)

$$\xi = 1 \rightarrow \psi = 0. \quad (26)$$

Taking this into account when combining (24) and (25) and introducing the dimensionless group

$$F = \frac{R}{L} \cdot \left(\frac{R}{2 \cdot L \cdot A} \right)^{1/3} \quad (27)$$

gives

$$\xi = \left\{ 1 - F \cdot \left[\frac{1}{6} \cdot \ln \frac{\psi^2 + \psi + 1}{(\psi - 1)^2} - \psi + \frac{1}{\sqrt{3}} \times \left(\arctan \frac{2 \cdot \psi + 1}{\sqrt{3}} - \arctan \frac{1}{\sqrt{3}} \right) \right] \right\}^{1/2}. \quad (28)$$

For a given value of F the dimensionless flow length can be calculated for any value of the relative film thickness ψ . The local relative thickness ψ can be calculated by iteration. Figure 5 shows some curves according to (28) for various values of F . With increasing values of F the relative film thickness decreases

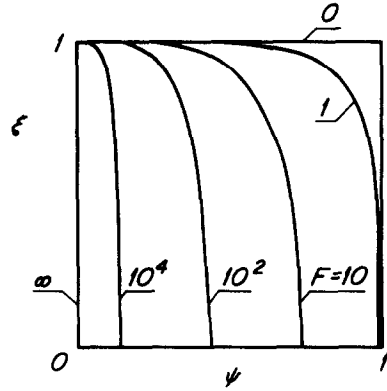


FIG. 5. Local relative film thickness according to (28) for various values of F .

and for $F = \infty$, $\psi = 0$. In the other limiting case of $F = 0$ the relative thickness $\psi = 1$. The film thickness is then constant over the surface which means also a constant heat flux for constant temperature difference. For the calculation of the mean heat-transfer resistance, as discussed before, and for the mean Nu number defined by (17) one needs the integrated value of ψ :

$$\psi_m = \int_{\xi=0}^{\xi=1} \psi \cdot d\xi = \int_{\psi=0}^{\psi=\psi_0} \xi \cdot d\psi. \quad (29)$$

This integral can be evaluated by the righthand integral if ψ_0 is that value for which

$$\xi(\psi_0) = 0. \quad (30)$$

Thus for any value of F the mean value ψ_m can be calculated by a numerical stepwise integration of (28)

according to (29). The maximum value of ψ_m is unity. Substituting ψ in (23) by $\psi_m = 1$ yields the maximum value of ϕ_m and thus with (17)

$$Nu_{min} = \left(\frac{2 \cdot L \cdot A}{R} \right)^{1/3} \quad (31)$$

The reciprocal value of ψ_m can then also be expressed by

$$\frac{1}{\psi_m} = \frac{Nu}{Nu_{min}} = u \quad (32)$$

which variable is only a function of F . Instead of the variable dimensionless group F one can also use the ratio of Nu_{∞} and Nu_{min} from (18) and (31).

$$\begin{aligned} \frac{Nu_{\infty}}{Nu_{min}} &= (0.86 \cdot A)^{1/4} \cdot \left(\frac{R}{2 \cdot L \cdot A} \right)^{1/3} \\ &= 0.809 \cdot F^{1/4} = v \end{aligned} \quad (33)$$

because it is only a function of F . The variables u and v have the advantage that u has the limiting values of $u = 1$ or $u = v$ for $v = 0$ or $v = \infty$, respectively, as shown in Fig. 6. For given values of L/R and A the

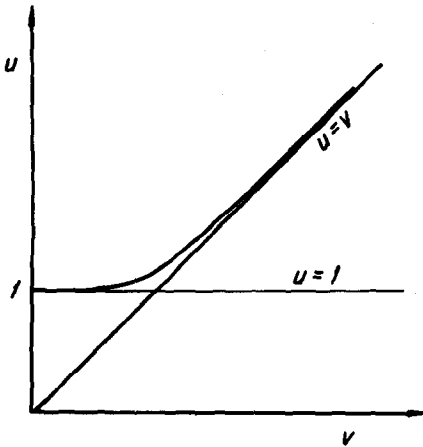


FIG. 6. Functional dependence of u on v according to (32) and (33).

value of v can be determined and from Fig. 6 the value of u and from u the Nu number can be determined. The curve u vs v in Fig. 6 which was determined by the numerical integrations described above can be approximated closely by

$$u = (1 + v^{31/7})^{7/31} \quad (34)$$

as shown in Table 1. Substituting u and v by the Nu ratios according to (32) and (33), solving for Nu and substituting Nu_{∞} and Nu_{min} according to (18) and (31) gives:

$$Nu = \left[\left(\frac{2 \cdot L \cdot A}{R} \right)^{31/21} + (0.86 \cdot A)^{31/28} \right]^{7/31} \quad (35)$$

Table 1. Dimensionless group u as function of v according to the numerical integration (32) and the approximation (34)

v , (33)	u , (32)	u , (34)	Relative error (%)
0.8036	1.0650	1.0754	+0.97
0.8928	1.1058	1.1128	+0.64
0.9541	1.1404	1.1437	-0.02
1.0000	1.1692	1.1694	+0.02
1.0305	1.1896	1.1877	-0.16
1.0995	1.2388	1.2323	-0.52
1.1439	1.2723	1.2632	-0.72
1.2099	1.3243	1.3117	-0.95
1.4387	1.5188	1.4991	-1.30
2.5584	2.5834	2.5673	-0.62
4.5495	4.5574	4.5508	-0.15
8.0903	8.0928	8.0905	-0.03

Thus the Nu number can be calculated for given values of L/R and A . For an infinite radius of curvature R (35) turns to (18). For a finite radius and with increasing values of A the first term in (35) becomes controlling and (35) approaches (31) with a constant film thickness. As (18) also (35) is applicable when the heat flux \dot{q} is given.

We now consider the case of a given mean temperature difference. Substituting Nu_{min} in (32) according to (31) and replacing A according to (21) gives

$$u = \left(\frac{R}{2 \cdot L \cdot E} \right)^{1/3} \cdot Nu^{4/3} \quad (36)$$

and now u is not proportional to Nu but to $Nu^{4/3}$.

Combining (21) and (33) yields accordingly

$$v = 0.764 \cdot \left(\frac{R}{L} \right)^{1/3} \cdot \left(\frac{Nu}{E} \right)^{1/12} \quad (37)$$

and v is now proportional to $Nu^{1/12}$. For a given pair L/R and E one can find the Nu number by trial and error. If u and v according to (36) and (37) fit the exact curve u vs v in Fig. 6, the estimated value of Nu was right. A fast converging method is to estimate a value of Nu and calculate v according to (37). Then a value of u can be found according to the curve in Fig. 6. With this value of u and (36) one can find an improved value of Nu with which to start again.

However, this method is very inconvenient and therefore a simple approximation is developed. Unfortunately, using (35) after substituting A according to (21) brings no advantage because the equation is then implicit in Nu and iterations are necessary. Only in the two limiting cases of Nu_{∞} and Nu_{min} can this equation be solved for Nu yielding in the first case Nu_{∞} according to (22) and in the other case

$$Nu = Nu_{min} = \left(\frac{2 \cdot L \cdot E}{R} \right)^{1/4} \quad (38)$$

This equation can be found through (36) with $u = 1$ or by combining (21) and (31). With the two limiting functions (22) and (38) the following equation similar to (35) was developed

$$Nu = \left[\left(\frac{2 \cdot L \cdot E}{R} \right)^{23/16} + (0.86 \cdot E)^{23/20} \right]^{4/23} \quad (39)$$

Thus the Nu number can also be directly determined when the ratio L/R and the value of the dimensionless group E can be calculated from the given data. The accuracy of (39) is about the same as that of (35) (see Table 1). In the region where both limiting functions are of the same order of magnitude, errors of about 1 per cent may arise. With respect to the curvature the validity range is estimated to be $0 \leq L/R \leq 0.2$.

In the limiting case $Nu = Nu_{\min}$ the Nu number is proportional to L , as follows from (5), (20), (31) and (38), and the heat-transfer coefficient is constant over the surface and independent of L .

5. NUMERICAL RESULTS AND CONCLUSIONS

To demonstrate the effect of curvature on the heat-transfer coefficient $\alpha = Nu \cdot \lambda/L$ some numerical results obtained from (35) and (39) are presented. We consider a section with a total flow length of $L = 1$ m and various small ratios of L/R . The circumferential velocity of the drum $\omega \cdot r = 10$ m/s and the radius $r = 0.75$ m. The saturation temperature of the steam inside the drum is taken as $T_s = 150^\circ\text{C}$ which corresponds to a pressure of $p_s = 4.76$ bar [6]. The driving mean temperature difference is assumed to be $\Delta T_m = 10^\circ\text{C}$. For simplification all properties are evaluated at the saturation temperature. With the properties according to [6] one finds that using (6) and (20) the value $E = 3.100 \times 10^{16}$. In Table 2, heat-transfer coefficients according to (38) and (39) are presented for various values of L/R . Further, the maximum absolute change of the drum radius Δr equal to the change of wall thickness is given in the table (for small values of L/R : $\Delta r/L = L/2R$). In Table 3, similar results are presented, however, for the case of a constant value of $A = 10^{13}$ using (31) and (35). The ratios α/α_∞ indicate the improvement in heat transfer due to the

Table 2. Heat-transfer coefficients for $E = 3.1 \times 10^{16}$ and various ratios of L/R

L/R	Δr (mm)	α_{\min} ($\text{W}/\text{m}^2 \cdot ^\circ\text{C}$)	α ($\text{W}/\text{m}^2 \cdot ^\circ\text{C}$)	α/α_∞	α/α_u
0	0	0	1319	1.00	1.55
0.001	0.5	1919	1956	1.48	2.30
0.002	1.0	2282	2299	1.74	2.70
0.005	2.5	2870	2876	2.18	3.37
0.010	5.0	3413	3416	2.59	4.01

Table 3. Heat-transfer coefficients for $A = 10^{13}$ and various ratios for L/R

L/R	Δr (mm)	α_{\min} ($\text{W}/\text{m}^2 \cdot ^\circ\text{C}$)	α ($\text{W}/\text{m}^2 \cdot ^\circ\text{C}$)	α/α_∞	α/α_u
0	0	0	1110	1.00	1.30
0.001	0.5	1857	1909	1.72	2.24
0.002	1.0	2339	2363	2.33	2.77
0.005	2.5	3175	3183	2.87	3.74
0.010	5.0	4000	4004	3.61	4.70

curvature. The ratios α/α_u indicate the improvement in the heat-transfer coefficient compared to the usual value at high speed drums being $852.15 \text{ W}/\text{m}^2 \cdot ^\circ\text{C}$ ($150 \text{ Btu}/\text{ft}^2 \cdot \text{h degF}$) according to Table 1 of [1].

The results of Tables 2 and 3 clearly demonstrate that even with very small curvatures and changes of wall thickness the heat-transfer coefficient increases considerably compared with the case of a cylindrical shape and, as expected, increases even more when compared with the usual cylindrical case with a rotating syphon.

Comparing α and α_{\min} reveals that in the region of remarkable heat-transfer improvement $\alpha \approx \alpha_{\min}$ and the local film resistance is practically constant. This gives together with the small change of wall thickness (compared to the mean thickness of a few centimetres) a virtually constant heat flux as desired for paper drying and as assumed for the derivation of (7).

Evaluating the heat-transfer resistance presented in Table 1 of [1] leads to the conclusion that by means of the condensate collecting groove and the curvature of the wall proposed in this paper, the overall heat-transfer coefficient could be increased by 20–40 per cent. The improvement would be even better when using a wall material with a higher conductivity.

The effect is influenced also by the length L or the number of sections applied in the drum.

6. EXTENSION TO OTHER CASES

The equations derived above are valid also for other cases in which the same type of differential equation (9) is valid.

Regarding a curved disk of the radius L and the radius of curvature R in a gravity field $b_n = g$ on which the condensate flows in radial direction yields the same differential equation. The only difference is a numerical factor in the dimensionless group A . Thus (35) and (39) are also valid for a curved disk in a gravity field when the following dimensionless groups are used:

$$A = \frac{b_n \cdot \rho \cdot h \cdot L^2}{3 \cdot v \cdot \dot{q}} \quad (40)$$

$$E = \frac{b_n \cdot \rho \cdot h \cdot L^3}{3 \cdot v \cdot \lambda \cdot \Delta T_m} \quad (41)$$

The same applies to a rotating flat disk in a coaxial gravity field (only radial flow neglecting coriolis acceleration), when the radius of curvature is replaced according to

$$R = \frac{b_n}{\omega^2}. \quad (42)$$

There are no restrictions with respect to the range of L/R . This case, however, without a gravity force normal to the disk, was treated by Sparrow and Gregg [7] and later by Dhir and Lienhard [8] yielding the limiting value Nu_{min} according to (38).

In [8] also a rotating plate shown in Fig. 7 was treated neglecting the forces normal to the wall. Also

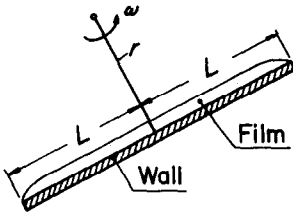


FIG. 7. Condensation on a rotating wall.

for this case our equations are valid using the original definitions (5) and (20) for A and E together with (6) and (42). Combining (6) and (42) gives

$$R = r \quad (43)$$

which can also be applied for any range of L/R . Using (6) implies that the coriolis acceleration is negligible. This is a good approximation because in the symmetrical case of Fig. 7 these forces have the opposite effect on the two flow paths of condensate. For high values of A and E the limiting value Nu_{min} is approached, which is the solution presented in [8].

The applications discussed above are based on the differential equation (9). In the following another extension of (35) and (39) is proposed which is not based on (9) but only on (7) and a more general function for b_x than (8). We make use of the fact that the limiting function Nu_{∞} according to (31) is the solution of (9) or (7) and (8) for the case of

$$b_n = 0 \quad (44)$$

which also compares with the results of [7] and [8]. Introducing (44) in (7) gives

$$0 = 2 \cdot b_x - \frac{b_n}{A} \cdot \frac{\xi}{\phi^3} \quad (45)$$

where b_n cancels against b_n in A . We now consider instead of (8) the more general function for the

acceleration in flow direction

$$b_x = b_{x1} \cdot \xi^a \quad (46)$$

where b_{x1} is the value at the end of the flow path. Substituting b_x in (45) according to (46) and solving for ϕ gives the local dimensionless film thickness.

$$\phi = \left(\frac{b_n}{2 \cdot A \cdot b_{x1}} \right)^{1/3} \cdot \xi^{(1-a)/3}. \quad (47)$$

By integrating ϕ one finds the mean film thickness. The reciprocal value according to (17) is then the limiting value Nu_{min} which can be expressed by (31) or (38) when the ratio L/R is replaced by

$$\frac{L}{R} = \left(\frac{4-a}{3} \right)^3 \cdot \frac{b_{x1}}{b_n}. \quad (48)$$

Introducing (48) also in (35) and (39) yields an approximation equation for the general case described by (46). This approach, however, should be subject to another more detailed investigation which is beyond the scope of this paper. Thus (35) and (39) can be used for quite a number of stationary or rotating condensing systems.

In all cases discussed above (39) provides a good approximation when applied to the corresponding cases of film evaporation and free convection, provided the dimensionless groups Gr and Pr according to (12) and (9) of [9] are introduced. When the Gr number is formed with b_n (not b as in [9]) (39) turns to

$$Nu = \left[\left(\frac{1}{3} \cdot \frac{L}{R} \cdot Gr_n \cdot Pr \right)^{23/16} + (0.143 \cdot Gr_n \cdot Pr)^{23/20} \right]^{4/23} \quad (49)$$

for the cases in which (5) and (20) are valid and to

$$Nu = \left[\left(\frac{2}{3} \cdot \frac{L}{R} \cdot Gr_n \cdot Pr \right)^{23/16} + (0.286 \cdot Gr_n \cdot Pr)^{23/20} \right]^{4/23} \quad (50)$$

when (40) and (41) have to be applied. Corresponding cases (in any acceleration field) are cooling on the upper side and heating on the underside of a plate. With curved walls the presented heat-transfer equations describe transfer on the convex side.

REFERENCES

1. S. T. Han, Drying of paper, *TAPPI* 53(6), 1034–1046 (1970).
2. W. Nusselt, Die Oberflächenkondensation des Wasserdampfes, *Z. Ver. Dt. Ing.* 60, 541–546, 569–575 (1916).
3. W. Roetzel, Heat transfer in laminar film condensation local film resistance and local enthalpy allowing for variable viscosity and subcooling, *Wärme- und Stoffübertragung* 6, 127–132 (1973).

4. I. Hütte, 28. Auflage, Wilhelm Ernst u. Sohn, Berlin (1955).
5. W. Meyer zur Capellen, *Integraltafeln*. Springer, Berlin (1950).
6. E. Schmidt, *VDI-Wasserdampfatafel*, 6. Auflage, Ausgabe B. Springer, Berlin (1963).
7. E. M. Sparrow and J. L. Gregg, A theory of rotating condensation, *J. Heat Transfer* **81**(2), 113–120 (1959).
8. V. Dhir and J. Lienhard, Laminar film condensation on plane and axisymmetric bodies in non-uniform gravity, *J. Heat Transfer* **93**(1), 97–100 (1971).
9. W. Roetzel, Gemeinsame Gleichungen für den Wärmeübergang bei laminarer freier Konvektion, Filmkondensation und Filmverdampfung, *Chemie-Ingr. Tech.* **43**(14), 785–791 (1971).

TRANSFERT THERMIQUE POUR DES TAMBOURS DE SECHAGE DE PAPIER, CHAUFFES A LA VAPEUR ET A ROTATION RAPIDE

Résumé—L'épaisseur du film d'un condensat et par conséquent le transfert thermique, dépend de la méthode d'enlèvement du condensat. Remplaçant le syphon rotatif conventionnel par un autre stationnaire avec l'ailette se déplaçant dans une gorge circonférentielle, la perte de pression élevée dans le syphon rotatif est supprimée tandis que l'épaisseur du film est sensiblement réduite.

On peut obtenir un accroissement plus important du transfert de chaleur en donnant une légère pente à la surface interne du tambour. Dans le cas où cette surface conique est circulairement incurvée dans la direction axiale, on établit des équations pour le calcul des épaisseurs locale et moyenne du film. On montre que pour un flux thermique uniforme donné, même avec des courbures très légères, les coefficients de transfert du côté de la vapeur sont plusieurs fois plus grand que ceux sur une surface cylindrique.

Les équations développées sont utiles aussi pour d'autres systèmes de condensation où l'accélération le long de l'écoulement de condensat est proportionnelle à la longueur d'écoulement. Les équations peuvent être appliquées de même aux cas de l'évaporation en convection libre et en film.

VERBESSERUNG DES WÄRMEÜBERGANGS IN DAMPFBEHEIZTEN SCHNELL ROTIERENDEN PAPIERTROCKENTROMMELN

Zusammenfassung—Die Kondensatfilmdicke und damit der Wärmeübergang hängen von der Art der Kondensatabfuhr ab. Das konventionelle rotierende Kondensatabflußrohr wird durch ein feststehendes ersetzt, dessen Mündung in eine umlaufende Sammelrinne hineinragt. Hierdurch wird der hohe Druckverlust im rotierenden Rohr vermieden und gleichzeitig die Filmdicke etwas verringert.

Eine entscheidende Erhöhung des Wärmeübergangs kann jedoch dadurch erreicht werden, daß die Innenfläche schwach konisch ausgeführt wird. Für den Fall, daß diese konische Innenwand in axialer Richtung kreisförmig gekrümmt ist, werden Gleichungen zur Berechnung der örtlichen und mittleren Filmdicke angegeben. Diese zeigen, daß schon bei sehr schwachen Krümmungen der Wärmeübergangskoeffizient innen gegenüber dem zylindrischen Fall bei praktisch konstantem Wärmefluß vervielfacht werden kann.

Die entwickelten Gleichungen sind auch brauchbar für andere Kondensationsvorgänge, bei denen die Beschleunigung in Kondensatflußrichtung proportional dem Strömungsweg ist. Die Gleichungen sind auch für entsprechende Fälle von freier Konvektion und Filmverdampfung anwendbar.

ИНТЕНСИФИКАЦИЯ ТЕПЛОБМЕНА В НАГРЕВАЕМЫХ ПАРОМ СКОРОСТНЫХ БАРАБАНАХ ДЛЯ СУШКИ БУМАГИ

Аннотация — Толщина пленки конденсата и, следовательно, теплоперенос зависят от способа удаления конденсата. Замена обычного вращающегося сифона с конденсатом неподвижным сифоном с наконечником, который движется в канавке на внутренней поверхности барабана, позволила устранить большой перепад давления при некотором снижении толщины пленки.

Более значительное увеличение теплопереноса можно получить, если внутренней поверхности барабана придать небольшой наклон. Представлены уравнения для расчёта локальной и средней толщины пленки для случая, когда коническая поверхность располагается в осевом направлении барабана. Эти уравнения показывают, что для требуемого однородного теплового потока даже при очень небольшой конусности коэффициенты теплопереноса со стороны пара могут увеличиться в несколько раз в сравнении с теплообменом в случае цилиндрической поверхности.

Полученные уравнения могут быть применены к другим системам с конденсатом, когда ускорение вдоль направления потока конденсата пропорционально длине потока. Эти уравнения могут быть также применены для соответствующих случаев свободной конвекции и испарения пленки.